

## QCD sum rules for $\Delta$ isobar in nuclear matter

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### Abstract

The self-energies of  $\Delta$  isobar propagating in nuclear matter are calculated using the finite-density QCD sum-rule methods. The calculations show that the Lorentz vector self-energy for the  $\Delta$  is significantly smaller than the nucleon vector self-energy. The magnitude of the  $\Delta$  scalar self-energy is larger than the corresponding value for the nucleon, which suggests a strong attractive net self-energy for the  $\Delta$ ; however, the prediction for the scalar self-energy is very sensitive to the density dependence of certain in-medium four-quark condensate. Phenomenological implications for the couplings of the  $\Delta$  to the nuclear scalar and vector fields are briefly discussed.

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The finite-density QCD sum-rule approach provides a framework to test the predictions of relativistic nuclear phenomenology for baryon self-energies in nuclear matter. It has been shown recently in Refs. [1–5] that the predictions of QCD sum rules for the nucleon self-energies are consistent with those obtained from relativistic phenomenological models (e.g., the relativistic optical potentials of Dirac phenomenology [6,7] or Brueckner calculations [8,9]). (Other applications of sum rule methods to finite density problems are discussed in Refs. [10–15]). In Refs. [16,17], the self-energies of the  $\Lambda$  and  $\Sigma$  hyperons are investigated within the same framework. The sum-rule calculations indicate that the self-energies of the  $\Sigma$  are close to the corresponding nucleon self-energies while the self-energies of the  $\Lambda$  are only about  $\frac{1}{3}$  of the nucleon self-energies. The sum-rule predictions for the baryon scalar self-energies are, however, sensitive to assumptions made about the density dependence of certain four-quark condensates [2,4,16,17]. In this brief report, we study the self-energies of the  $\Delta$  isobar in an infinite nuclear matter within finite-density QCD sum-rule approach.

Various investigators have discussed the roles of  $\Delta$  in the hadronic field theories [18–22]. In these relativistic models,  $\Delta$  is treated as stable particle, which couples to the same scalar and vector fields as the nucleon, but with different strengths. Many interesting physical results depend on the choice of the coupling strengths [18–20,22]. However, little is known about these coupling strengths. The vector coupling for the  $\Delta$  is expected to be similar to the corresponding coupling of the nucleon based on the SU(6) symmetry [23,20]. A weak restriction can also be obtained if one demands that no real  $\Delta$ 's are present in the ground state of nuclear matter at saturation density [20],  $r_s \leq 0.82r_v + 0.71$ , where  $r_s(r_v)$  is the ratio of the scalar (vector) coupling for  $\Delta$  to that for the nucleon. The finite-density sum rules may offer new information on these coupling strengths.

We find that the  $\Delta$  vector self-energy is significantly smaller than the sum-rule prediction for the nucleon vector self-energy. In terms of an effective theory of baryons and mesons, this implies a much smaller vector coupling for the  $\Delta$  than would be expected from SU(6) symmetry. The predictions for the  $\Delta$  scalar self-energy are very sensitive to the assumed density dependence of the four-quark condensate  $\langle \bar{q}q \rangle_{\rho_N}^2$ . If we assume that  $\langle \bar{q}q \rangle_{\rho_N}^2$  depends weakly on the nucleon density (such that the nucleon sum rules predict a strong attractive scalar self-energy which cancels the nucleon vector self-energy [2,4]), then the magnitude of the  $\Delta$  scalar self-energy is found to be larger than the corresponding value for the nucleon and the net  $\Delta$  self-energy is strong and attractive. If we assume that  $\langle \bar{q}q \rangle_{\rho_N}^2$  has a strong dependence on the nucleon density (in this case the nucleon scalar self-energy is very small and the net nucleon self-energy is large and repulsive [2,4]), the  $\Delta$  scalar self-energy is very small and the net  $\Delta$  self-energy is moderate and repulsive.

To derive the finite-density sum rules for  $\Delta$ , we start from the correlator defined by

$$\Pi_{\mu\nu}^{\Delta}(q) \equiv i \int d^4x e^{iq \cdot x} \langle \Psi_0 | T[\eta_{\mu}^{\Delta}(x) \bar{\eta}_{\nu}^{\Delta}(0)] | \Psi_0 \rangle, \quad (1)$$

where  $\eta_{\mu}^{\Delta}(x)$  is a colorless interpolating field, constructed from quark fields, which carries the quantum numbers of  $\Delta$  isobar. The ground state of nuclear matter  $|\Psi_0\rangle$  is characterized by the nucleon density  $\rho_N$  in the rest frame and the nuclear matter four-velocity  $u^{\mu}$ ; it is assumed to be invariant under parity and time reversal. Here we consider the interpolating fields that do not contain derivatives. The interpolating field for  $\Delta$  is then unique and can be written as [24]

$$\eta_\mu^\Delta(x) = \varepsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]u_c(x), \quad (2)$$

where T denotes a transpose in Dirac space,  $C$  is the charge conjugation matrix,  $a$ ,  $b$ , and  $c$  are color indices.

The correlator  $\Pi_{\mu\nu}^\Delta(q)$  can have a number of distinct structures [24]. However, the three structures proportional to  $g_{\mu\nu}$ ,  $g_{\mu\nu}\not{q}$ , and  $g_{\mu\nu}\not{u}$  receive contributions from spin  $\frac{3}{2}$  states only (see Refs. [24,25])

$$\Pi_{\mu\nu}^\Delta(q) \equiv \Pi_s(q^2, q_0)g_{\mu\nu} + \Pi_q(q^2, q \cdot u)g_{\mu\nu}\not{q} + \Pi_u(q^2, q \cdot u)g_{\mu\nu}\not{u} + \dots \quad (3)$$

So, we will focus on the three invariant functions,  $\Pi_s$ ,  $\Pi_q$  and  $\Pi_u$ , which are functions of the two Lorentz scalars  $q^2$  and  $q \cdot u$ . In the zero-density limit,  $\Pi_u \rightarrow 0$ , and  $\Pi_s$  and  $\Pi_q$  become functions of  $q^2$  only. For convenience, we will work in the rest frame of nuclear matter hereafter, where  $u^\mu = (1, \mathbf{0})$  and  $qu \rightarrow q_0$ ; we also take  $\Pi_i(q^2, qu) \rightarrow \Pi_i(q_0, |\mathbf{q}|)$  ( $i = \{s, q, u\}$ ). To obtain QCD sum rules, we need to construct a phenomenological representation for  $\Pi_{\mu\nu}^\Delta(q)$  and to evaluate  $\Pi_{\mu\nu}^\Delta(q)$  using operator product expansion (OPE) techniques.

The analytic structure of the correlator  $\Pi_{\mu\nu}^\Delta$ , and consequently the invariant functions  $\Pi_s$ ,  $\Pi_q$  and  $\Pi_u$  is revealed by a standard Lehmann representation in energy variable  $q_0$ , at fixed three momentum  $\mathbf{q}$  [2]

$$\Pi_i(q_0, |\mathbf{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\mathbf{q}|)}{\omega - q_0} + \text{polynomial} \quad (4)$$

for each invariant function  $\Pi_i$ ,  $i = \{s, q, u\}$ . The polynomial stands for contributions from the contour at large  $|q_0|$ , which will be eliminated by a subsequent Borel transform (see below). The discontinuity  $\Delta\Pi_i$  (which is the spectral density up to a constant) defined by,  $\Delta\Pi_i(\omega, |\mathbf{q}|) \equiv \lim_{\epsilon \rightarrow 0^+} [\Pi_i(\omega + i\epsilon, |\mathbf{q}|) - \Pi_i(\omega - i\epsilon, |\mathbf{q}|)]$ , contains the spectral information on the quasiparticle, quasihole, and higher energy states.

At finite density, the spectral densities for baryon and antibaryon are not simply relate because the ground state is no longer invariant under ordinary charge conjugation. Here we assume that a quasiparticle description of the  $\Delta$  is reasonable. In the context of relativistic phenomenology, the  $\Delta$  is assumed to couple to the same scalar and vector fields as the nucleons in the nuclear matter, and is treated as quasiparticle with real Lorentz scalar and vector self-energies. We follow Refs. [2,4,16,17] and assume a pole ansatz for the quasibaryon (higher-energy states are included in a continuum contribution), which, in the Rarita-Schwinger formalism [26], can be expressed as [20,25]

$$\Pi_{\mu\nu}^\Delta = -\lambda_\Delta^{*2} \frac{\not{q} + M_\Delta^* - \not{u}}{(q_0 - E_q)(q_0 - \overline{E}_q)} [g_{\mu\nu} + \dots] , \quad (5)$$

where the ellipses denote the other distinct structures. This implies [2]

$$\Delta\Pi_s(\omega, |\mathbf{q}|) = +2\pi i \frac{M_\Delta^* \lambda_\Delta^{*2}}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \overline{E}_q)] , \quad (6)$$

$$\Delta\Pi_q(\omega, |\mathbf{q}|) = +2\pi i \frac{\lambda_\Delta^{*2}}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \overline{E}_q)] , \quad (7)$$

$$\Delta\Pi_u(\omega, |\mathbf{q}|) = -2\pi i \frac{\Sigma_v \lambda_\Delta^{*2}}{2E_q^*} [\delta(\omega - E_q) - \delta(\omega - \overline{E}_q)] , \quad (8)$$

where  $\lambda_\Delta^{*2}$  is an overall residue. Here we have defined  $M_\Delta^* \equiv M_\Delta + \Sigma_s$ ,  $E_q^* \equiv \sqrt{M_\Delta^{*2} + \mathbf{q}^2}$ ,  $E_q \equiv \Sigma_v + \sqrt{M_\Delta^{*2} + \mathbf{q}^2}$ , and  $\overline{E}_q \equiv \Sigma_v - \sqrt{M_\Delta^{*2} + \mathbf{q}^2}$ , where  $M_\Delta$  is the mass of  $\Delta$  and  $\Sigma_s$  and  $\Sigma_v$  are identified as the scalar and vector self-energies of a  $\Delta$  in nuclear matter. The positive- and negative-energy poles are at  $E_q$  and  $\overline{E}_q$ , respectively.

The OPE for the correlator can be carried out using the simple rules and techniques outlined in Refs. [3,5]. We work to leading order in perturbation theory. Contributions proportional to the up and down current quark masses and the terms proportional to the condensate  $\langle (\alpha_s/\pi) [(u \cdot G)^2 + (u \cdot \tilde{G}^2)]_{\rho_N}$  are neglected as they give numerically small contributions [4,5]. We consider quark and quark-gluon condensate up to dimension five and pure gluon condensates of dimension four. At dimension six, we include only the four-quark condensates.

The QCD sum rules follow by equating the spectral representation of the correlator to the corresponding OPE representation. We observe that a negative-energy pole, occurring at  $\overline{E}_q$ , is introduced in Eqs. (6)–(8). This corresponds to an antiparticle in the nuclear matter. Since we want to focus on the positive-energy quasiparticle pole, we follow Refs. [2,4,16,17] and construct the sum rules that suppress the contributions from the region of the negative energy excitations:

$$\mathcal{B}[\Pi_i^E(q_0^2, |\mathbf{q}|) - \overline{E}_q \Pi_i^O(q_0^2, |\mathbf{q}|)]_{\text{QCD}} = \mathcal{B}[\Pi_i^E(q_0^2, |\mathbf{q}|) - \overline{E}_q \Pi_i^O(q_0^2, |\mathbf{q}|)]_{\text{phen}} , \quad (9)$$

for  $i = \{s, q, u\}$ , where the left-hand side is obtained from the OPE and right-hand side from the phenomenological dispersion relations. Here the operator  $\mathcal{B}$  is defined in Ref. [4], and  $\Pi_i^E$  and  $\Pi_i^O$  are the even and odd pieces in  $q_0$  of the invariant functions:

$$\Pi_i(q_0, |\mathbf{q}|) = \Pi_i^E(q_0^2, |\mathbf{q}|) + q_0 \Pi_i^O(q_0^2, |\mathbf{q}|) , \quad (10)$$

for  $i = \{s, q, u\}$ .

With the spectral ansatz of Eqs. (6)–(8) and our calculations from the OPE, we obtain the following sum rules for the  $\Delta$ :

$$\begin{aligned}
\lambda_{\Delta}^{*2} M_{\Delta}^* e^{-(E_q^2 - \mathbf{q}^2)/M^2} &= -\frac{M^4}{3\pi^2} E_1 \langle \bar{q}q \rangle_{\rho_N} L^{16/27} + \frac{M^2}{6\pi^2} E_0 \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N} L^{4/27} \\
&\quad - \frac{M^2}{36\pi^2} \left( 7E_0 + 32 \frac{\mathbf{q}^2}{M^2} \right) \left( \langle \bar{q} i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{8} \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \right) L^{4/27} \\
&\quad + \frac{4}{3} \bar{E}_q \langle \bar{q}q \rangle_{\rho_N} \langle q^\dagger q \rangle_{\rho_N} L^{16/27} , \tag{11}
\end{aligned}$$

$$\begin{aligned}
\lambda_{\Delta}^{*2} e^{-(E_q^2 - \mathbf{q}^2)/M^2} &= \frac{M^6}{80\pi^4} E_2 L^{4/27} + \frac{M^2}{6\pi^2} E_0 \bar{E}_q \langle q^\dagger q \rangle_{\rho_N} L^{4/27} \\
&\quad - \frac{5}{9} \frac{M^2}{32\pi^2} E_0 \langle \frac{\alpha_s}{\pi} G^2 \rangle_{\rho_N} L^{4/27} \\
&\quad - \frac{M^2}{9\pi^2} \left( E_0 - 4 \frac{\mathbf{q}^2}{M^2} \right) \langle q^\dagger i D_0 q \rangle_{\rho_N} L^{4/27} \\
&\quad - \frac{2\bar{E}_q}{3\pi^2} \left( 1 - \frac{\mathbf{q}^2}{M^2} \right) \left( \langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{12} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \right) L^{4/27} \\
&\quad + \frac{4}{3} \langle \bar{q}q \rangle_{\rho_N}^2 L^{28/27} + \frac{2}{3} \langle q^\dagger q \rangle_{\rho_N}^2 L^{4/27} , \tag{12}
\end{aligned}$$

$$\begin{aligned}
\lambda_{\Delta}^{*2} \Sigma_v e^{-(E_q^2 - \mathbf{q}^2)/M^2} &= \frac{M^4}{4\pi^2} E^2 \langle q^\dagger q \rangle_{\rho_N} L^{4/27} + \frac{8M^2}{9\pi^2} E_0 \bar{E}_q \langle q^\dagger i D_0 q \rangle_{\rho_N} L^{4/27} \\
&\quad - \frac{31M^2}{144\pi^2} E_0 \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} L^{4/27} \\
&\quad + \frac{M^2}{6\pi^2} \left( E_0 + 10 \frac{\mathbf{q}^2}{M^2} \right) \left( \langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{12} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \right) L^{4/27} \\
&\quad + \frac{4}{3} \bar{E}_q \langle q^\dagger q \rangle_{\rho_N}^2 L^{4/27} , \tag{13}
\end{aligned}$$

where  $L \equiv \ln(M/\Lambda_{\text{QCD}})/\ln(\mu/\Lambda_{\text{QCD}})$ . We take  $\mu = 0.5 \text{ GeV}$  and  $\Lambda_{\text{QCD}} = 0.1 \text{ GeV}$ . Here we have adopted the notations of Ref. [16] and defined  $E_0 \equiv 1 - e^{-s_0^*/M^2}$ ,  $E_1 \equiv 1 - e^{-s_0^*/M^2} \left( \frac{s_0^*}{M^2} + 1 \right)$ , and  $E_2 \equiv 1 - e^{-s_0^*/M^2} \left( \frac{s_0^{*2}}{2M^4} + \frac{s_0^*}{M^2} + 1 \right)$ , which account for continuum corrections to the sum rules, where  $s_0^* = \omega_0^2 - \mathbf{q}^2$  is the continuum threshold. We use a universal effective threshold for simplicity. In our calculations, we have ignored the anomalous dimensions of dimension four and five operators.

For dimension three and four in-medium condensates, we use the values quoted in Ref. [16]. For dimension five condensates, we take  $\langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N} = \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle_{\text{vac}} + (0.62 \text{ GeV}^2) \rho_N$  [3],  $\langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} = (-0.33 \text{ GeV}^2) \rho_N$  [27],  $\langle \bar{q} i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{8} \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N} = (0.085 \text{ GeV}^2) \rho_N$  [3], and  $\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \frac{1}{12} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} = (0.031 \text{ GeV}^2) \rho_N$  [3]. Four-quark condensates are numerically important in both the vacuum and the finite-density  $\Delta$  sum rules. In the sum rules derived above, we included the contributions from the four-quark condensates in their in-medium factorized forms [3]; however, the factorization approximation may not be justified in nuclear matter. Thus, we follow Ref. [4] and parametrize the scalar-scalar four-quark condensate so that it interpolates between its factorized form in free space and its factorized form in nuclear matter:

$$\langle \bar{q}q \rangle_{\rho_N}^2 \longrightarrow \langle \widetilde{\bar{q}q} \rangle_{\rho_N}^2 \equiv (1-f) \langle \bar{q}q \rangle_{\text{vac}}^2 + f \langle \bar{q}q \rangle_{\rho_N}^2 , \tag{14}$$

where  $f$  is a real parameter. The density dependence of the scalar-scalar four-quark condensate is now parametrized by  $f$  and the density dependence of  $\langle \bar{q}q \rangle_{\rho_N}$ . The factorized condensate  $\langle \bar{q}q \rangle_{\rho_N}^2$  appearing in Eq. (12) will be replaced by  $\langle \widetilde{\bar{q}q} \rangle_{\rho_N}^2$  in the calculations to follow. The other four-quark condensates give small contributions. So, we use their factorized form for simplicity. All the finite-density results presented are obtained at the nuclear matter saturation density, which is taken to be  $\rho_N = (110 \text{ MeV})^3$ .

To extract the self-energies from the sum rules, we use the same procedure as used in Refs. [2,4,16,17]. To get a prediction for the  $\Delta$  mass, we apply the same procedure to the sum rules evaluated in the zero-density limit. We follow Ref. [2] and rely on the cancellation of systematic discrepancies by normalizing finite-density predictions for all self-energies to the zero-density prediction for the mass. We choose a fixed Borel window at  $1.05 \leq M \leq 1.6 \text{ GeV}$  in our analysis. The study of the  $\Delta$  sum rules in vacuum suggests that the sum rules are valid in this region [24].

In Fig. 1, we displayed the optimized results for the ratios  $M_\Delta^*/M_\Delta$  and  $\Sigma_v/M_\Delta$  as functions of  $f$  for  $|\mathbf{q}| = 270 \text{ MeV}$ . One notices that  $\Sigma_v/M_\Delta$  is not sensitive to  $f$ , and the sum rule prediction is

$$\Sigma_v/M_\Delta \simeq 0.09\text{--}0.11. \quad (15)$$

The finite-density nucleon sum rules predict  $\Sigma_v/M_N \simeq 0.24\text{--}0.37$  [4]. Thus, we find  $(\Sigma_v)_\Delta/(\Sigma_v)_N \sim 0.4\text{--}0.5$ . This result, if interpreted in terms of a relativistic hadronic model, would imply that the coupling of the  $\Delta$  to the Lorentz vector field is much weaker than the corresponding nucleon coupling. This compares to the SU(6) expectation of 1.

The ratio  $M_\Delta^*/M_\Delta$ , however, varies rapidly with  $f$ . Therefore, the sum-rule prediction for the scalar self-energy is very sensitive to the density dependence of the scalar-scalar four-quark condensate. For small values of  $f$  ( $0 \leq f \leq 0.3$ ), the predictions are

$$M_\Delta^*/M_\Delta \simeq 0.62\text{--}0.71, \quad (16)$$

which implies  $\Sigma_s/M_\Delta \simeq -(0.29\text{--}0.38)$ . With the nucleon sum-rule prediction  $M_N^*/M_N \simeq 0.63\text{--}0.72$ , we obtain  $(\Sigma_s)_\Delta/(\Sigma_s)_N \sim 1.3$ . In a hadronic model, this implies a stronger coupling of the  $\Delta$  to the Lorentz scalar field than for nucleon. In this case, the net  $\Delta$  self-energy is strong and attractive. For large values of  $f$  ( $f \sim 1$ ), the predictions turn out to be  $M_\Delta^*/M_\Delta \sim 1$ , which implies a very weak scalar self-energy and a sizable repulsive net self-energy for the  $\Delta$ .

In conclusion, we have studied the self-energies of  $\Delta$  isobar in nuclear matter using the finite-density QCD sum-rule methods. The sum-rule calculations indicate that the  $\Delta$  vector self-energy is much smaller than the corresponding nucleon self-energy. In terms of a relativistic hadronic model, this result implies that the vector coupling for the  $\Delta$  is significantly smaller than the corresponding nucleon coupling to the vector meson ( $r_v \sim 0.4\text{--}0.5$ ). The sum-rule prediction for the  $\Delta$  scalar self-energy is somewhat indefinite as the predictions are sensitive to the undetermined density dependence of four-quark condensates. If the four-quark condensates only depends weakly on the nucleon density (so that the sum-rule predictions for the *nucleon* self-energies are consistent with known relativistic phenomenology), we find a large and attractive scalar self-energy for the  $\Delta$ , the magnitude of which is larger than the value for the nucleon ( $r_s \sim 1.3$ ). In this case, the net self-energy for  $\Delta$  is

strong and attractive. Clearly, phenomenological constraints on the density dependence of the four-quark condensates from other sources will be very important. Work in this direction is in progress [28].

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## FIGURES

FIG. 1. Optimized sum-rule predictions for  $M_{\Delta}^*/M_{\Delta}$  and  $\Sigma_v/M_{\Delta}$  as functions of  $f$ , with  $|\mathbf{q}| = 270 \text{ MeV}$ . The other input parameters are described in the text.

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